

Effective use of mathematical equations in an online learning environment

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Mathematical analysis is an essential tool in the practice of science, technology, engineering, and mathematics, and consequently it is important for students in these subjects to demonstrate effective use of mathematics (Goldstone & Landy, 2009). In this talk we are interested in supporting the use of mathematical equations in an online learning environment; in particular, we require methods of supporting both the entry and automated marking of mathematical equations, in order to support immediate personalised feedback to the learner.

We report on our experience designing, building, and using Inequality: an open-source formula entry tool which works across all major browsers, supports both mouse and touch-based entry, and is usable by high school children and teachers. Inequality is composed of a graphical, drag-and-drop front-end interface to build expressions in response to a question and a back-end service. It automatically marks answers entered with model answers for the given question as specified by our team of content creators, with various degrees of flexibility in how two expressions are considered equivalent.

Inequality has been used for nearly three years to support over 20,000 students and 900 teachers of GCSE and A level Physics. Since May 2019, Inequality supported over 300 pupils and about 50 teachers with symbolic Boolean expressions as taught in A level Computer Science.

We describe the effect Inequality has on the behaviour and performance of students using our learning platform. We compared the behaviour of students who used approximately

350 physics and mathematics questions, in either multiple choice or symbolic format. Our analysis explored nearly 500,000 answer attempts and determined that 73% of the 350 questions required fewer attempts to answer correctly in symbolic format. Because the Boolean logic questions in our computer science platform were developed symbolically from the beginning, we currently cannot perform the same comparison.

We also looked at how formulae are constructed using Inequality across physics and computer science. We built action trees comprising actions such as dragging a symbol from the menu, attaching and detaching a symbol to another symbol, and so on. We found that, while there are a few recurring and efficient ways of building correct answers, many students arrived at correct answers in less efficient and sometimes more convoluted ways. For example, some students effectively used Inequality to manipulate an expression as they built it, adapting what they would otherwise do with pen and paper. We found very similar patterns in both physics and computer science, which tells us that students work in similar ways, and suggests that many of the benefits seen in the physics platform may translate to the computer science platform.

We asked some of our students from the physics platform to complete a feedback questionnaire that was essential in contextualising our quantitative analysis. We analysed 563 valid responses from students who were largely in Year 12 (typically aged 16–17) and started using the physics platform in the same year. The questionnaires produced three key findings:

1. Students found it more difficult to work with larger formulae – mainly due to some usability issues that we since fixed – but they generally do not avoid work that they think will require large formulae
2. Students have a slight preference towards Inequality as opposed to pen and paper when they need to manipulate formulae, and they find the editor helpful in working out solutions
2. Students do not find Inequality distracting or hindering in their workflow

References

- Franceschini, A., Sharkey, J.P. & Beresford, A.R. (2019) Inequality: Multi-modal equation entry on the web. Proceedings of the Sixth ACM Conference on Learning @ Scale, 1–10. Available from: <https://doi.org/10.1145/3330430.3333625>
- Goldstone, R. & Landy, D. (2009) How much of symbolic manipulation is just symbol pushing? Proceedings of the Annual Meeting of the Cognitive Science Society, 31.



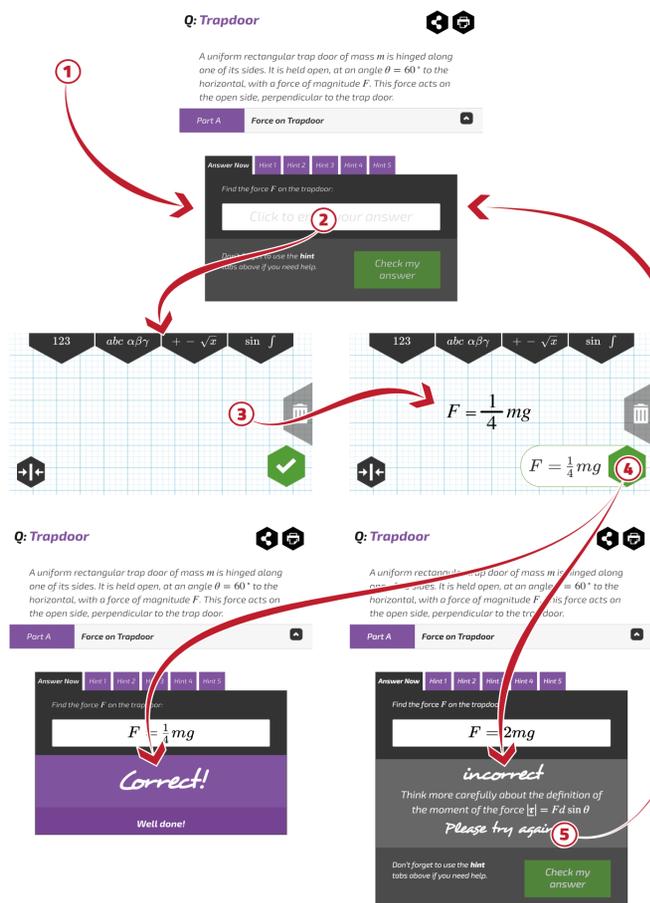
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We developed a free, STEM oriented, online platform supporting students and teachers in secondary education. The platform revolves around automatically marked questions of various types: multi-choice, numeric, text-based, symbolic. Symbolic manipulation is an important skill in many STEM subjects, but traditional computer-based symbolic entry systems can be inaccessible to our target audience.

We developed *Inequality*, a symbolic system composed of

1. a web-based, graphical, cross-platform editor that allows students to work with familiar symbolic notation, and
2. a service that marks entered answers using model answers specified by our content creators.

We have two instances of our platform: **Isaac Physics** covers **Physics** and **Mathematics**, and **Isaac Computer Science** includes **Boolean Logic** questions which use a similar notation to mathematics. The picture below shows an example of the question-answering workflow for a Physics question.



We analysed data from Isaac Physics as it has been running for longer than Isaac Computer Science, and has a dataset of nearly **500 000** question attempts. A preliminary analysis of the Computer Science dataset suggests that many of the results from the Physics platform can translate to Computer Science.

We looked at three key aspects:

1. the **performance** of students while answering questions;
2. how the students **use Inequality** to answer questions;
3. the **experience** of students using Inequality.

1. Performance

Many symbolic questions in Isaac Physics started their life as numeric questions. In this format, students were required to construct a formula and use it to compute a numeric answer. We converted 343 questions to the symbolic format. Students were equally proficient at answering questions correctly in both formats, however students required **fewer attempts** when answering questions using *Inequality* **73% of the time**. The Boolean logic questions on Isaac Computer Science started as symbolic questions so we cannot repeat this analysis, but we hypothesise students will experience a similar benefit..

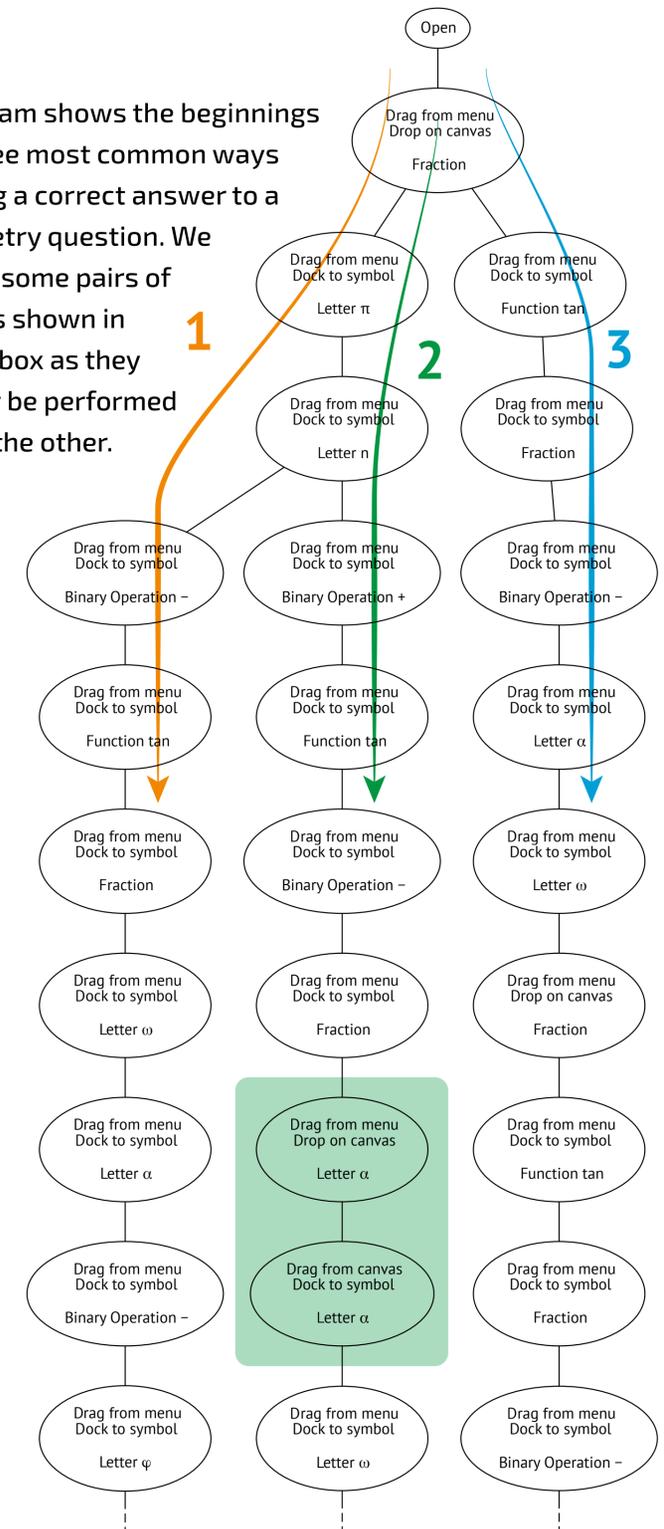
2. Usage

We looked at the way students use *Inequality* to answer questions. To do this, we created action trees from our system logs to explore every step taken by students in formula construction. An example is shown to the right. This analysis highlighted a few efficient and commonly chosen means of constructing expression for any given question. However, it also showed that many students like to perform manipulations in the editor, for example re-arranging a formula to reach the correct answer. The key finding is that *Inequality* enables a **high level of flexibility** while working with symbolic notation.

3. Experience

Students responding to a questionnaire reported being generally OK with *Inequality* despite a few usability issues which we have since fixed. We think that its **flexibility** is a key part in this result.

This diagram shows the beginnings of the three most common ways of building a correct answer to a trigonometry question. We combined some pairs of actions, as shown in the green box as they could only be performed one after the other.



References

A. Franceschini, J. P. Sharkey, A. R. Beresford (2019) *Inequality: multi-modal equation entry on the web*. In Proceedings of Learning @ Scale 2019, Chicago, IL, USA.